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Dynamics of structural models with a long-range interaction: Glassy versus nonglassy behavior

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By making use of the Langevin dynamics and its generating functional (GF) formulation, the influence of the long-range nature of the interaction on the tendency of the glass formation is systematically investigated. In doing so, two types of models are considered: (i) the nondisordered model with a pure repulsive type of interaction, and (ii) the model with a randomly distributed strength of interaction (a quenched disordered model). The long-ranged potential of interaction is scaled with a number of particles N in such a way as to enable for the GF the saddle-point treatment as well as the systematic $1/N$ expansion around it. We show that the nondisordered model has no glass transition, which is in line with the mean-field limit of the mode-coupling theory (MCT) predictions. On the other hand, the model with a long-range interaction that has a quenched disorder leads to MC equations which are generic for the p-spin glass model and polymeric manifold in a random media.

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I. INTRODUCTION

The theoretical description of slow dynamics is a crucial point to elucidate the nature of the glass transition in structural glass-forming liquids. One of the commonly used approaches, mode-coupling theory (MCT), was designed from the very beginning for the supercooled simple liquids [1], i.e., for the nondisordered models (as opposed to the models which contain quenched disorder naturally). Later it was proven that MC equations become exact for a number of spin-glass models [2–8] as well as for the polymeric manifold in a random media [7–11] (i.e., for the models with quenched disorder) provided that the number of variable components goes to the infinite.

Actually, applicability of the MC equations has been substantially extended to the case when the time translation invariance and the fluctuation-dissipation theorem does not hold any more [8]. This striking similarity between the models with and without quenched disorder suggests that the effective disordered potential (e.g., in a supercooled liquid) is in a sense ‘‘ self-induced’’ and the difference between such a ‘‘self-induced disorder’’ and the quenched disorder might not be crucial [7,8].

In order to provide some insight into self-induced disorder, we employed in Ref. [12] a Feynman variational principle (VP) for a set of interacting particles. Indeed it was shown that the VP is capable of treating metastable states of the glass-forming system. The main point in Ref. [12] was that the partition function representation in terms of functional integrals is twofold: (a) either as an integral over the local density, $\rho(\mathbf{r})$, or (b) over the conjugated to $\rho(\mathbf{r})$ field $\psi(\mathbf{r})$. It has been *assumed* that the *component average* free energy $\overline{F}$ (which is only meaningful in the supercooled regime) is equal to the variational free energy $F_{\rm VP}$. There are at least four strong reasons in favor of this (at first sight not obvious) assumption.

(i) The variational free energy $F_{\rm VP}$ is an upper bound for the *canonical* free energy, i.e., $F_c \leqslant \overline{F} = F_{\rm VP}$, as it should be since $F_c = \overline{F} - T\Sigma$, where the complexity $\Sigma \geqslant 0$ [13].

(ii) After implementation of VP, the initial problem is reduced to a self-consistent random field Ginzburg-Landau model (RFGLM). Then, as was shown previously, the corresponding field $\psi(\mathbf{r})$ must be upgraded to a *replicated* field $\psi_a(\mathbf{r})$, where $a = 1, \ldots, n$ (with the final limit $n \to 0$) and the density field $\rho(\mathbf{r})$ plays the role of an ‘‘external’’ field. Note that here the density $\rho(\mathbf{r})$ is Gaussian due to the use of the VP. Eventually the correlators of ψ and ρ fields can be determined self-consistently.

(iii) The resulting replicated partition function for RFGLM has a typical form which may eventually lead to the replica symmetry breaking (RSB), structural glass transition, and the ‘‘self-induced’’ disorder.

(iv) Finally, in the case of the long-range interaction, the partition function allows the expansion around the saddle point, or mean-field (MF), solution. It is possible to show then that the *next to the mean-field approximation* and VP merge and both become exact, i.e., $\overline{F} = F_c$ and the glassy phase does not appear.

Some evidence for this behavior was deduced from the results for the particles on an M- dimensional hypersphere [14] at large dimensions, $M \to \infty$.

The aim of this paper is to face the full dynamical problem for a nondisorded model with a long-range interaction. Using the expansion around the saddle-point solution, we derive the full equation of motion for the time-dependent density-density correlator and show that a ‘‘glassy’’ solution does not exist. Conversely, if we add a term describing quenched disorder, by random distribution of the strength of the interaction potential, then the resulting equations of motion for two time density correlation and response functions fall in the same class as MC equations which have been widely discussed [2–11]. This means that the ‘‘self-induced’’ disorder is not generic for the pure model with the long-range interaction, and conversely on addition of a quenched disorder the phase space becomes very rugged resulting in slow dynamical processes.

The paper is organized as follows. In Sec. II we introduce the theoretical model without quenched disorder. Its dynamics is discussed by using the functional integral technique. The saddle-point solution yields the mean-field dynamics. Expansions around the saddle point yield one-loop corrections. The Legendre transformation provides the possibility of the analysis of the full dynamic correlation matrix. In Sec. III, quenched disorder is introduced by a ''random bond model'' and a Gaussian disorder. The corresponding generating functional (GF) is computed by the self-consistent Hartree approximation, which results in a set of coupled Langevin equations, which are solved in their asymptotic regimes. More details on the calculations are laid out in the corresponding Appendixes.

II. THE MODEL WITHOUT QUENCHED DISORDER

We start from a simple model system which consists of interacting particles. To do so, let us consider a set of $N(\gg 1)$ particles in d-dimensional space interacting by a pair potential of the form

$$V(r)=\left(\frac{\mu}{N}\right)\frac{\exp(-\kappa r)}{4\pi r^{\alpha}}. \tag{2.1}$$

This is a typical example of a long-range potential with a characteristic length κ and a coupling constant μ/N. The choice of this potential is twofold. It contains a cutoff at κ^{-1} and thus allows to control the range of the interaction. Moreover, at small scales ($r<\kappa^{-1}$) it consists of a typical power-law decay with long-range character, if $0<\alpha<2$. Therefore, the so chosen potential allows us to keep control of the range and nature of the interaction, which will become essential below. To ensure extensivity of the total interaction energy, we require that the integral $\int d^d V(\mathbf{r})=\mathcal{O}(N^0)$, i.e., it does not depend on the number of particles N. As a result, we have $\kappa\propto N^{-1/(d-\alpha)}$. The intermolecular potential (2.1) has the form of the generalized Kac potential,

$$V(\mathbf{r})=\kappa^d f(\kappa\mathbf{r}), \tag{2.2}$$

which has been used for the rigorous treatment of the van-der-Waals theory [15]. In order to provide conditions for the expansion around a saddle point, carried out later on (see below), we should require that the length κ^{-1} must be larger compared to the characteristic size of the system (which scales naturally as $N^{1/d}$) at $N\to\infty$. As a consequence, we find the limits for the range parameter α,

$$0<\alpha<d. \tag{2.3}$$

Below we shall restrict our considerations to the case $d=3,\alpha=1$, and the strength of the interaction $\mu>0$ (pure repulsion) without loss of generalization in the main statements that we are going to predict. Then the Fourier transformation of the potential (2.1) takes the especially simple form

$$V(\mathbf{k})=\left(\frac{\mu}{N}\right)\frac{1}{k^2+\kappa^2}, \tag{2.4}$$

which allows accurate analytic calculations. In the limit $N\to\infty$ we have thus $\kappa^2\propto N^{-1}$, but the relevant minimum wave vector is $k_{\min}^2\propto N^{-2/3}$ and thus κ^2 can be actually neglected under the integration over the whole k space. As a result, we arrive formally at a one-component plasma model (OCP) [16] where the electroneutrality is implicitly provided by a neutralizing background.

A. The generating functional method

In the following we set up the relevant equations of motion for the model system. We restrict ourselves to the Langevin dynamics, which can be comfortably formulated in terms of dynamic functionals, which allows the systematic $1/N$-expansion treatment. The Langevin dynamics of N particles interacting via the potential (2.1) (at $d=3,\alpha=1$, and $\mu>0$) is described by the equation of motion

$$m_0\frac{\partial^2}{\partial t^2}\mathbf{r}^{(p)}(t)+\gamma_0\frac{\partial}{\partial t}\mathbf{r}^{(p)}(t) -\frac{\mu}{N}\sum_{m=1}^{N}\nabla v(\mathbf{r}^{(p)}-\mathbf{r}^{(m)})=\mathbf{f}^{(p)}(t), \tag{2.5}$$

where m_0 and γ_0 are the mass and the friction coefficient, respectively, $p=1,2,\ldots,N$, and $v(r;\kappa)=\exp(-\kappa r)/4\pi r$. The random force in Eq. (2.5) is Gaussian with $\langle f_i^{(p)}(t)\rangle=0$ and the correlator

$$\langle f_i^{(p)}(t)f_j^{(n)}(t')\rangle=2T\gamma_0\delta_{pm}\delta_{ij}\delta(t-t'), \tag{2.6}$$

where from now on we work in units where the Boltzmann constant $k_{\rm B}=1$.

As was mentioned, it is more convenient to reformulate the Langevin problem (2.5) and (2.6) by using the celebrated Martin-Siggia-Rose generating functional (GF) method [17]. The method was first applied for the ϕ^4 model with the long-range interaction in [18] and for the polymer melt dynamics in [19,20]. Despite the fact that the Langevin equation (2.5) is of the second order, it is possible to show that the Jacobian which appears under transformation to the functional variables is still equal to one (see the Appendix in [21]). After using this technique for the problem (2.5) and (2.6), the GF takes the form

$$Z\{\cdots\}=\int\prod_{p=1}^{N}D\mathbf{r}^{(p)}(t)D\hat{\mathbf{r}}^{(p)}(t)\exp\left\{\sum_{p=1}^{N}A_0[\mathbf{r}^{(p)},\hat{\mathbf{r}}^{(p)}] +\int dt\sum_{p=1}^{N}\sum_{m=1}^{N}\frac{\mu}{N}i\hat{r}_j^{(p)}(t)\nabla_j^{(p)}v(\mathbf{r}^{(p)}-\mathbf{r}^{(m)})\right\}, \tag{2.7}$$

where the action of the free system

$$A_0[\mathbf{r}^{(p)},\hat{\mathbf{r}}^{(p)}]=\int dt\left\{T\gamma_0[i\hat{r}_j^{(p)}(t)]^2+i\hat{r}_j^{(p)}(t)\left[m_0\frac{\partial^2}{\partial t^2}r_j^{(p)}(t)+\gamma_0\frac{\partial}{\partial t}r_j^{(p)}(t)\right]\right\}. \tag{2.8}$$

In the following we are going to transform this functional to collective density variables. By using the transformations to the mass density

$$\rho(\mathbf{r})=\sum_{p=1}^{N}\delta(\mathbf{r}-\mathbf{r}^{(p)}(t)) \tag{2.9}$$

and the longitudinal projection of the response field density

$$\pi(\mathbf{r})=\sum_{p=1}^{N}i\hat{r}_i^{(p)}(t)\nabla_i\delta(\mathbf{r}-\mathbf{r}^{(p)}(t)) \tag{2.10}$$

for the GF one gets

$$Z\{\chi_\alpha\}=\int\prod_{\alpha=0}^{1}D\rho_\alpha(1)\exp\left\{W\{\rho_\alpha\}-\frac{1}{2}\int d1d2\,\rho_\alpha(1)U_{\alpha\beta}(1,2)\rho_\beta(2)+\int d1\,\rho_\alpha(1)\chi_\alpha(1)\right\}, \tag{2.11}$$

where the summation over the repeated Greek indices is implied. In Eq. (2.11) we have introduced the two-dimensional field

$$\rho_\alpha(1)\equiv\begin{pmatrix}\rho(1)\\ \pi(1)\end{pmatrix}, \tag{2.12}$$

where $\alpha=0,1$ and $1\equiv(\mathbf{r},t)$. The ''entropy'' of the free system is given as usual by

$$W\{\rho,\pi\}=\ln\int\prod_{p=1}^{N}D\mathbf{r}^{(p)}(t)D\hat{\mathbf{r}}^{(p)}(t)\exp\left\{\sum_{p=1}^{N}A_0\{\mathbf{r}^{(p)},\hat{\mathbf{r}}^{(p)}\}\right\}\times\delta\left[\rho(\mathbf{r},t)-\sum_{p=1}^{N}\delta(\mathbf{r}-\mathbf{r}^{(p)}(t))\right]\times\delta\left[\pi(\mathbf{r},t)-\sum_{p=1}^{N}i\hat{r}_j^{(p)}(t)\nabla_j\delta(\mathbf{r}-\mathbf{r}^{(p)}(t))\right], \tag{2.13}$$

$U_{\alpha\beta}$ is the 2×2-interaction matrix

$$U_{\alpha\beta}(1,2)=\begin{pmatrix}0 & V(|\mathbf{r}_1-\mathbf{r}_2|)\\ V(|\mathbf{r}_1-\mathbf{r}_2|) & 0\end{pmatrix}, \tag{2.14}$$

and $\chi_\alpha(1)$ is a source field.

An alternative valuable representation of the GF can be obtained through the ''functional Fourier transformation''

$$\exp\{F\{\psi_\alpha\}\}=\int D\rho_\alpha(1)\exp\left\{W\{\rho_\alpha\}-i\int d1\,\rho_\alpha(1)\psi_\alpha(1)\right\} \tag{2.15}$$

and its inversion

$$\exp\{W\{\rho_\alpha\}\}=\int D\psi_\alpha(1)\exp\left\{F\{\psi_\alpha\}+i\int d1\,\rho_\alpha(1)\psi_\alpha(1)\right\}. \tag{2.16}$$

The substitution of Eq. (2.13) into Eq. (2.15) leads to the explicit expression for the free-system GF,

$$\exp\{F\{\psi_\alpha\}\}=\int\prod_{p=1}^{N}D\mathbf{r}^{(p)}(t)D\hat{\mathbf{r}}^{(p)}(t)\times\exp\left\{\sum_{p=1}^{N}A_0[\mathbf{r}^{(p)},\hat{\mathbf{r}}^{(p)}]-i\sum_{p=1}^{N}\int dt\,\psi(\mathbf{r}^{(p)})+i\sum_{p=1}^{N}\int dt\,i\hat{r}_j^{(p)}(t)\nabla_j\phi(\mathbf{r})\Big|_{\mathbf{r}=\mathbf{r}^{(p)}(t)}\right\}, \tag{2.17}$$

where $\psi(1)$ and $\phi(1)$ are components of the column variable

$$\psi_\alpha(1)\equiv\begin{pmatrix}\psi(1)\\ \phi(1)\end{pmatrix}. \tag{2.18}$$

By making use of Eq. (2.16) in Eq. (2.11) and after functional integration over $\rho_\alpha(1)$, one gets

$$Z\{\chi_\alpha,\lambda_\alpha\}=\int\prod_{\alpha=0}^{1}D\psi_\alpha(1)\exp\left\{F\{\psi_\alpha\}+\frac{1}{2}\int d1d2[i\psi_\alpha(1)+\chi_\alpha(1)][U^{-1}]_{\alpha\beta}(1,2)\times[i\psi_\beta(2)+\chi_\beta(2)]+\int d1\,\psi_\alpha(1)\lambda_\alpha(1)\right\}, \tag{2.19}$$

where we have also added a source field $\lambda_\alpha(1)$ conjugated to $\psi_\alpha(1)$. As a result, Eqs. (2.11) and (2.19) provide two equivalent representations of the GF. For the purpose of expansion around the saddle point, we use representation (2.19) at $\lambda_\alpha(1)=0$, which after the transformation $\psi_\alpha\rightarrow\psi_\alpha+i\chi_\alpha$ yields

$$Z\{\chi_\alpha\}=\int\prod_{\alpha=0}^{1}D\psi_\alpha(1)\exp\{-NA[\psi_\alpha;\chi_\alpha]\}, \tag{2.20}$$

which is appropriate for a saddle-point integration, since the particle number N is large. The action hereby is given as

$$A[\psi_\alpha;\chi_\alpha]=\frac{1}{2}\int dt\int d^3\mathbf{r}d^3\mathbf{r}'\,\psi_\alpha(\mathbf{r},t)[v^{-1}]_{\alpha\beta}\times(\mathbf{r}-\mathbf{r}';\kappa)\psi_\beta(\mathbf{r}',t)-\frac{1}{N}\ln\int\prod_{p=1}^{N}D\mathbf{r}^{(p)}(t)\times D\hat{\mathbf{r}}^{(p)}(t)\exp\Bigg\{\sum_{p=1}^{N}A_0[\mathbf{r}^{(p)},\hat{\mathbf{r}}^{(p)}]-i\sum_{p=1}^{N}\int dt r_\alpha^{(p)}(t)[\psi_\alpha(\mathbf{r}^{(p)}(t))+i\chi_\alpha(\mathbf{r}^{(p)}(t))]\Bigg\},\tag{2.21}$$

and the interaction matrix

$$v_{\alpha\beta}(\mathbf{r};\kappa)=\begin{pmatrix}0&1\\1&0\end{pmatrix}\frac{\exp(-\kappa r)}{4\pi r}.\tag{2.22}$$

Recall that the relation $\kappa\propto N^{-1/2}$ is necessary for the validity of the saddle-point integration. Moreover, we have defined the column vector

$$r_\alpha^{(p)}(t)=\begin{pmatrix}1\\-i\hat{r}_j^{(p)}(t)\int d\tau\dfrac{\delta}{\delta r_j^{(p)}(\tau)}\end{pmatrix}\tag{2.23}$$

for convenience.

B. The saddle-point solution and expansion around the SP

Minimization of $A[\psi_\alpha;\chi_\alpha]$ with respect to $\psi_\alpha(1)$ leads to the SP equations for the mean fields $\overline{\psi_\alpha}(1)$,

$$\overline{\psi_\alpha}(\mathbf{r},t)=-\frac{i\mu}{N}\int d^3\mathbf{r}'\,v_{\alpha\beta}(\mathbf{r}-\mathbf{r}')\langle\rho_\beta(\mathbf{r}',t)\rangle_{\mathrm{SP}},\tag{2.24}$$

where the average $\langle\cdots\rangle_{\mathrm{SP}}$ is calculated by using the *cumulant* GF,

$$P_{\mathrm{SP}}\{\overline{\psi_\alpha}+i\chi_\alpha\}\equiv\frac{1}{N}\ln\int\prod_{p=1}^{N}D\mathbf{r}^{(p)}(t)D\hat{\mathbf{r}}^{(p)}(t)\times\exp\Bigg\{\sum_{p=1}^{N}A_0[\mathbf{r}^{(p)},\hat{\mathbf{r}}^{(p)}]-i\sum_{p=1}^{N}\int dt r_\alpha^{(p)}(t)[\overline{\psi_\alpha}(\mathbf{r}^{(p)}(t))+i\chi_\alpha(\mathbf{r}^{(p)}(t))]\Bigg\}.\tag{2.25}$$

The correlation matrix in the random-phase approximation (RPA) is defined in such a way that

$$S_{\alpha\beta}(1,2)=\lim_{\overline{\psi_\alpha}+i\chi_\alpha\to0}\left[\frac{\delta\langle\rho_\alpha(1)\rangle_{\mathrm{SP}}}{N\delta\chi_\beta(2)}\right].\tag{2.26}$$

After linearization of Eq. (2.25) with respect to $\overline{\psi_\alpha}+i\chi_\alpha$, the 2×2 RPA correlation matrix is easily found to coincide with the well-known form [19]

$$S_{\alpha\beta}(1,2)=\{[\hat{F}^{-1}+\mu\hat{v}]^{-1}\}_{\alpha\beta}(1,2),\tag{2.27}$$

where $\hat{v}$ is the interaction matrix (2.22) and $F_{\alpha\beta}$ is the correlation matrix for the *free system*. $F_{\alpha\beta}(1,2)=\langle\Delta\rho_\alpha(1)\Delta\rho_\beta(2)\rangle_0/N$ has the form

$$F_{\alpha\beta}(1,2)=\begin{pmatrix}F_{00}(1,2)&F_{01}(1,2)\\F_{10}(1,2)&0\end{pmatrix}.\tag{2.28}$$

In Eq. (2.28), $F_{01}(1,2)$ and $F_{10}(1,2)$ are response functions whereas $F_{00}(1,2)$ stands for the correlation function. The relation between them is given by the fluctuation dissipation theorem (FDT), which in $(\mathbf{k},t)$ representation has the form

$$-\beta\frac{\partial}{\partial t}F_{00}(\mathbf{k},t)=F_{01}(\mathbf{k},t)-F_{10}(\mathbf{k},t).\tag{2.29}$$

It is easy to check that in this case the FDT for the RPA-type correlation matrix (2.27) also holds,

$$-\beta\frac{\partial}{\partial t}S_{00}(\mathbf{k},t)=S_{01}(\mathbf{k},t)-S_{10}(\mathbf{k},t),\tag{2.30}$$

where $\beta=1/T$ is the inverse temperature. The corresponding elements of the RPA matrix (2.27) are of an especially simple form in the Fourier $(\mathbf{k},\omega)$ representation, namely

$$S_{00}(\mathbf{k},\omega)=\frac{F_{00}(\mathbf{k},\omega)}{[1+\mu v(k)F_{10}(\mathbf{k},\omega)][1+\mu v(k)F_{01}(\mathbf{k},\omega)]},\tag{2.31}$$

$$S_{01}(\mathbf{k},\omega)=\frac{F_{01}(\mathbf{k},\omega)}{1+\mu v(k)F_{01}(\mathbf{k},\omega)},\tag{2.32}$$

$$S_{10}(\mathbf{k},\omega)=\frac{F_{10}(\mathbf{k},\omega)}{1+\mu v(k)F_{10}(\mathbf{k},\omega)}.\tag{2.33}$$

It turns out to be interesting to recover the well-known form in the static limit, where we have $S_{01}(\mathbf{k},\omega\to0)=\beta S_{\mathrm{st}}(\mathbf{k})=[(\beta F_{\mathrm{st}})^{-1}+\mu k^{-2}]$, and for the correlator $S_{\mathrm{RPA}}(\mathbf{k})=S_{\mathrm{st}}(\mathbf{k})/\rho_0$ one gets

$$S_{\mathrm{RPA}}(\mathbf{k})=\frac{1}{1+\dfrac{\beta\mu\rho_0}{k^2}},\tag{2.34}$$

where we have used $F_{\mathrm{st}}=\rho_0$. This expression is completely equivalent to the correlator for the OCP model [see Eq. (10.1.7) in [16]] with the direct correlation function $c(\mathbf{k})=-\mu\beta/k^2$ and the Debye wave number $k_D=(\beta\mu\rho_0)^{1/2}$.

Now let us expand the action (2.21) around the SP solution (2.24) up to the second order with respect to the fluctuations $\psi_\alpha(1)-\overline{\psi_\alpha}(1)$. After the functional integration, we arrive at the following result for the GF:

$$P\{\chi_\alpha\} \equiv \frac{1}{N} \ln Z\{\chi_\alpha\}$$

$$= P_{\rm SP}\{\overline{\psi}_\alpha + \chi_\alpha\} - \frac{1}{2N} {\rm Tr}[\ln T_{\alpha\beta}(1,2)], \quad (2.35)$$

where $T_{\alpha\beta}(1,2)$ is the inverse matrix of the effective interactions [24],

$$T_{\alpha\beta}(1,2) = \frac{1}{\mu}[v^{-1}]_{\alpha\beta}(1,2) + \frac{1}{N}\langle \Delta\rho_\alpha(1)\Delta\rho_\beta(2)\rangle_{\rm SP}. \quad (2.36)$$

In Eqs. (2.35) and (2.36) we deliberately keep the external field $\chi_\alpha(1)$ nonzero because it is to be used in the next subsection for the Legendre transformation.

C. The Legendre transformation

The functional Legendre transformation is a general way to provide the Dyson equation for the *full correlation matrix* $G_{\alpha\beta}(1,2)$ [22]. In doing so, the *irreducible* GF, $\Gamma\{\langle\rho_\alpha(1)\rangle\}$, is defined by the identity

$$\Gamma\{\langle\rho_\alpha(1)\rangle\} + P\{\langle\chi_\alpha(1)\rangle\} = \int d1 \langle\rho_\alpha(1)\rangle\chi_\alpha(1). \quad (2.37)$$

By doing functional differentiation of Eq. (2.37), one gets

$$\chi_\alpha(1) = \frac{\delta\Gamma\{\langle\rho_\alpha(1)\rangle\}}{\delta\langle\rho_\alpha(1)\rangle} \quad (2.38)$$

and

$$[G^{-1}]_{\alpha\beta}(1,2) = \frac{\delta^2\Gamma\{\langle\rho_\alpha(1)\rangle\}}{\delta\langle\rho_\alpha(1)\rangle\delta\langle\rho_\beta(2)\rangle}. \quad (2.39)$$

Taking into account the result in Eq. (2.35), we find the following result for GF:

$$\Gamma\{\langle\rho_\alpha(1)\rangle\} = \Gamma_{\rm SP}\{\langle\rho_\alpha(1)\rangle\} + \frac{1}{2N}{\rm Tr}[\ln T_{\alpha\beta}(1,2)], \quad (2.40)$$

where

$$\Gamma_{\rm SP}\{\langle\rho_\alpha(1)\rangle\} = -P_{\rm SP}\{\chi_\alpha\} + \int d1\langle\rho_\alpha(1)\rangle\chi_\alpha(1). \quad (2.41)$$

In Eq. (2.40) one should consider $\chi_\alpha(1)$ as a functional of $\langle\rho_\alpha(1)\rangle$ given by Eq. (2.38). Double differentiation of Eq. (2.40) leads to an equation of the Dyson form,

$$[G^{-1}]_{\alpha\beta}(1,2) = [S^{-1}]_{\alpha\beta}(1,2) - \Sigma_{\alpha\beta}(1,2), \quad (2.42)$$

where the RPA-correlation matrix, $S_{\alpha\beta}(1,2)$, is defined by Eqs. (2.31)–(2.33) and the ''self-energy'' functional $\Sigma_{\alpha\beta}(1,2)$ has the form

$$\Sigma_{\alpha\beta}(1,2) = -\frac{1}{2N}{\rm Tr}\left\{\frac{\delta^2}{\delta\langle\rho_\alpha(1)\rangle\delta\langle\rho_\beta(2)\rangle}\ln T_{\gamma\delta}(3,4)\right\}_{\chi_\alpha=0}. \quad (2.43)$$

In Eq. (2.43), the ''trace'' is taken over the variables 3,4 and indices γ,δ. The explicit differentiation in Eq. (2.43) leads to the result

$$\Sigma_{\alpha\beta}(1,2) = -\frac{1}{2N}{\rm Tr}\left\{\hat{T}^{-1}\frac{\delta^2\hat{T}}{\delta\langle\rho_\alpha(1)\rangle\delta\langle\rho_\beta(2)\rangle}\right\}_{\chi_\alpha=0}, \quad (2.44)$$

where $\hat{T}$ is a short-hand notation of the matrix $T_{\gamma\delta}(3,4)$ and we have taken into account that $\delta T_{\alpha\beta}(1,2)/\delta\chi_\gamma(3) = \langle\Delta\rho_\alpha(1)\Delta\rho_\beta(2)\Delta\rho_\gamma(3)\rangle_{\rm SP}/N = 0$ at $\chi_\alpha=0$ because the fluctuations are Gaussian. Further calculation yields

$$\frac{\delta^2 T_{\gamma\delta}(3,4)}{\delta\langle\rho_\alpha(1)\rangle\delta\langle\rho_\beta(2)\rangle} = \int d5d6\frac{1}{N}\langle\Delta\rho_\gamma(3)\Delta\rho_\delta(4)\Delta\rho_\omega(5)\Delta\rho_\chi(6)\rangle_{\rm SP} \times R_{\omega\beta}(5,2)R_{\chi\alpha}(6,1), \quad (2.45)$$

where

$$R_{\alpha\beta}(1,2) = \frac{\delta\vartheta_\alpha(1)}{\delta\langle\rho_\beta(2)\rangle} \quad (2.46)$$

and the full mean field

$$\vartheta_\alpha(1) = -i\overline{\psi}_\alpha(1) + \chi_\alpha(1). \quad (2.47)$$

The expression for $R_{\alpha\beta}(1,2)$ can be easily found by differentiation of Eq. (2.47) with respect to $\langle\rho_\beta(2)\rangle$. Taking into account Eqs. (2.24), (2.38), and (2.39) at $\chi_\alpha\to 0$ one gets

$$R_{\alpha\beta}(1,2) = [G^{-1}]_{\alpha\beta}(1,2) - \mu\int d4d3\, v_{\alpha\omega}(1,4)S_{\omega\gamma}(4,3)R_{\gamma\beta}(3,2) \quad (2.48)$$

or finally

$$R_{\alpha\beta}(1,2) = \int d3\{[\hat{1} + \mu\hat{v}\hat{S}]^{-1}\}_{\alpha\gamma}(1,3)[\hat{G}^{-1}]_{\gamma\beta}(3,2), \quad (2.49)$$

where the hatted variables stand for the corresponding 2×2 matrices. Substitution of Eqs. (2.49) and (2.45) in Eq. (2.44) yields

$$\Sigma_{\alpha\beta}(1,2) = -\int d3d4K_{\gamma\delta}(3,4)[G^{-1}]_{\gamma\alpha}(3,1) \times [G^{-1}]_{\delta\beta}(4,2), \quad (2.50)$$

where the 2×2 vertex matrix has the form

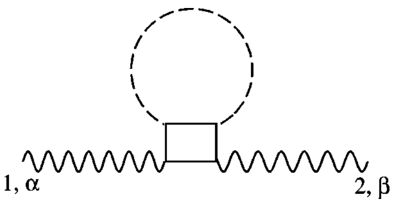

FIG. 1. Diagramatic interpretation of the vertex matrix: the rectangle corresponds to $S^{(4)}_{\alpha\beta\gamma\delta}(1,2,3,4)$; the dashed line to the effective interaction matrix $[(\mu\hat{v})^{-1}+\hat{S}]^{-1}$; the wavy line to $[\hat{1}+\mu\hat{v}\hat{S}]^{-1}$.

$$K_{\alpha\beta}(1,2)=\int d3d4d5d6\{[(\mu\hat{v})^{-1}+\hat{S}]^{-1}\}_{\delta\gamma}\times(4,3)S^{(4)}_{\gamma\delta\omega\chi}(3,4,5,6)\{[\hat{1}+\mu\hat{v}\hat{S}]^{-1}\}_{\omega\alpha}\times(5,1)\{[\hat{1}+\mu\hat{v}\hat{S}]^{-1}\}_{\chi\beta}(6,2). \tag{2.51}$$

In Eq. (2.51), $S^{(4)}_{\alpha\beta\gamma\delta}(1,2,3,4)$ is the four-point (response) correlator matrix in the RPA,

$$S^{(4)}_{\alpha\beta\gamma\delta}(1,2,3,4)=\frac{1}{N^2}\langle\Delta\rho_\alpha(1)\Delta\rho_\beta(2)\Delta\rho_\gamma(3)\Delta\rho_\delta(4)\rangle_{\mathrm{SP}}. \tag{2.52}$$

The explicit calculation of $S^{(4)}_{\alpha\beta\gamma\delta}(1,2,3,4)$ is implemented in Appendix B. The vertex matrix can be seen as a one-loop diagram (see Fig. 1).

The higher-loop contributions, which include generally speaking $2m$-point correlators, $S^{(2m)}_{\alpha\beta\ldots\gamma}(1,2,\ldots,2m)$, can be also considered, however the ''self-energy'' still has the same convolution structure : $\hat{\Sigma}=\hat{G}^{-1}*\tilde{K}*\hat{G}^{-1}$. Here the vertex matrix $\tilde{K}_{\alpha\beta}(1,2)$ is calculated in the RPA only. That is why these contributions basically do not change our results.

As a result, in the $(\mathbf{k},\omega)$ representation the Dyson equation (2.42) with the ''self-energy'' functional (2.50) reduces to a quadratic one,

$$G_{\alpha\gamma}(\mathbf{k},\omega)[S^{-1}]_{\gamma\delta}(\mathbf{k},\omega)G_{\delta\beta}(\mathbf{k},\omega)-G_{\alpha\beta}(\mathbf{k},\omega)+K_{\alpha\beta}(\mathbf{k},\omega)=0. \tag{2.53}$$

The coefficients of Eq. (2.53) trace the problem back to the *free system dynamics*, which is embodied in the correlation matrices $F_{\alpha\beta}(1,2)$ and $F^{(4)}_{\alpha\beta\gamma\delta}(1,2,3,4)$. It is not surprising then that a specification of the model for the free system dynamics is necessary, before going to the investigation of Eq. (2.53).

D. Analysis of the equation for the full correlation matrix

As we have mentioned, the explicit solution of Eq. (2.53) needs the specification of the free system dynamics. Two simple models are most amenable for the theoretical treatment: the free diffusion model (FDM) and the relaxation time approximation model (RTAM) [16,23]. The latter provide more reasonable dynamical information also for short time intervals, $\Delta t < m_0/\gamma_0$, where the FDM completely failed (e.g., the sum rule does not hold). It turns out that upon calculation of the trace in Eq. (2.51), the integral is ultraviolet-divergent for FDM and only RTAM leads to the finite result. The matrix elements for RTAM have the form

$$F_{00}(\mathbf{k},\omega)=\frac{2F_{\mathrm{st}}k^2D}{\omega^2+(k^2D-\omega^2\tau_0)^2}, \tag{2.54}$$

$$F_{01}(\mathbf{k},\omega)=\frac{\beta F_{\mathrm{st}}k^2D}{-i\omega+k^2D-\omega^2\tau_0}, \tag{2.55}$$

$$F_{10}(\mathbf{k},\omega)=\frac{\beta F_{\mathrm{st}}k^2D}{i\omega+k^2D-\omega^2\tau_0}, \tag{2.56}$$

where we introduced the diffusion coefficient $D=T/\gamma_0$, the characteristic time scale $\tau_0=m_0/\gamma_0$, and $F_{\mathrm{st}}=\rho_0$ for the overall density. At $\tau_0=0$ we return to FDM. In the case of RTAM the solution of Eq. (2.53) for the full correlation matrix reads

$$G_{01}(\mathbf{k},\omega)=\frac{1+\sqrt{1-4[-i\omega\tau_c-\omega^2\tau_0\tau_c+\chi_{\mathrm{st}}^{-1}]K_{01}(\mathbf{k},\omega)}}{2[-i\omega\tau_c-\omega^2\tau_0\tau_c+\chi_{\mathrm{st}}^{-1}(k)]}, \tag{2.57}$$

$$G_{10}(\mathbf{k},\omega)=G_{01}(-\mathbf{k},-\omega), \tag{2.58}$$

and

$$G_{00}(\mathbf{k},\omega)=\frac{\dfrac{\tau_c}{2\beta}\left|\dfrac{1+\sqrt{1-4[-i\omega\tau_c-\omega^2\tau_0\tau_c+\chi_{\mathrm{st}}^{-1}(\mathbf{k})]K_{01}(\mathbf{k},\omega)}}{-i\omega\tau_c-\omega^2\tau_0\tau_c+\chi_{\mathrm{st}}^{-1}(\mathbf{k})}\right|^2-K_{00}(\mathbf{k},\omega)}{\mathrm{Re}\{\sqrt{1-4[-i\omega\tau_c-\omega^2\tau_0\tau_c+\chi_{\mathrm{st}}^{-1}(\mathbf{k})]K_{01}(\mathbf{k},\omega)}\}}. \tag{2.59}$$

The explicit calculation of the matrix $K_{\alpha\beta}(\mathbf{k},\omega)$ [see Eq. (2.51)] is given in Appendix C. The overall behavior of the correlation function $G_{00}(\mathbf{k},\omega)$ according Eq. (2.59) is shown in Fig. 2 (at $\mu=10$, $\beta=0.1$, $\rho_0=1$, and $\tau_0=0.1$). It can be seen clearly there that no singularity appears at $\omega\to 0$.

The singularity, however, might be responsible for a glass transition. Instead, the low-frequency limit of $G_{00}(\mathbf{k},\omega)$ slowly changed with control parameters (which is not shown in Fig. 2). That means that for the nondisordered model with a general repulsive long-ranged potential (2.1) the glass transition is not generic. This very important conclusion suggests that for the model with a long-range interaction the phase

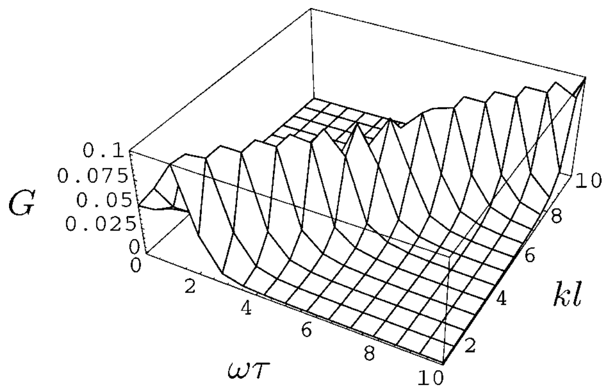

FIG. 2. The correlation function $G_{00}(\mathbf{k},\omega)$ vs rescaled variables $\omega\tau_0$ and kl_0, where $\tau_0=m_0/\gamma_0$ and $l_0=(\tau_0/\beta\gamma_0)^{1/2}$.

space is too smooth to show a glass transition. In order to obtain a glasslike transition, a competing interaction or a quenched disorder should be added. This leads to glassy dynamics, as we will show in the next section.

It is interesting to note that for the generalized Kac potential Eq. (2.2), where $f(\mathbf{r})$ and its Fourier transformation are positive definite functions, the MCT-memory kernel vanishes at $\kappa\to 0$ [25]. The corresponding argumentation is relegated to Appendix D. The explanation for this result lies in the fact that the ''cage effect,'' which is a cornerstone of MCT, is missing in the MF limit.

The ''glass transition'' which has been studied in Ref. [26] for the particles interacted via the Kac potential (2.2) has a completely different nature. In Ref. [26] the function $f(\mathbf{r})$ has a step form so that its Fourier transform $f(\mathbf{k})$ is negative at some value of $\mathbf{k}$. As a result, the system becomes unstable and a nonuniform configuration where the particles are grouped into ''clumps'' shows up. It was found that the slow dynamics of the MF model is associated with these clumps and does not touch a single-particle motion. Obviously, it is different from the conventional glass transition [1].

III. THE STRUCTURAL MODEL WITH COMPETING QUENCHED INTERACTIONS

A. Specification of the model

In the previous sections we have shown in detail that in the absence of disorder the dynamical spectrum changed monotonically with a control parameter and no glassy dynamics can be seen. The natural question which arises now is as follows: How will the introduction of competing interactions and/or quenched disorder affect the dynamics of the system discussed above? To provide an answer to this question, we will use already existing models of heteropolymers and their disordered two-body interaction [27–30]. The use of these models and techniques is natural here, since the behavior of heteropolymers is well discussed in the literature. In principle, two practical possibilities exist.

(i) The strength of the two-body interaction, μ, in Eq. (2.1) is now a random function of all pairs of the interacted particles, μ_{pm} (''random-bond model'').

(ii) Each particle carries a single ''charge'' σ_p, so that $\mu_{pm}=\xi_0+b\sigma_p\sigma_m$ and each σ_p is randomly distributed (''random sequence model'').

It turns out to be sufficient for the purpose of this paper to restrict ourselves only to the ''random-bond model,'' where μ_{pm} does not depend on the choice of pairs and has a Gaussian distribution,

$$P\{\mu_{pm}\}\propto\exp\left\{-\frac{(\mu_{pm}-\xi_0)^2}{2\chi^2}\right\}. \tag{3.1}$$

The competing long-range interactions frustrate the system of particles and the question is whether a glass transition exists or not. Normally, frustration and frozen disorder are enough for the existence of glassy phases. Here the problem is more complicated, since the long-range nature of the interaction may provide the opposite effects.

The averaging over the quenched disorder in Eq. (2.7) (after the substitution $\mu\to\mu_{pm}$) can be carried out in just the same way as in Refs. [31,32]. Similarly, typical two-time-dependent terms immediately appear. They are also bilinear with respect to the forces of interaction $\nabla_j v(\mathbf{r})$. In order to rationalize these terms, it is convenient to introduce [besides the mass density (2.9) and the response field density (2.10)] the following collective variables:

$$Q_0(\mathbf{r},t;\mathbf{r}',t')=\sum_{p=1}^{N}\delta(\mathbf{r}-\mathbf{r}^p(t))\delta(\mathbf{r}'-\mathbf{r}'^p(t')),$$

$$\begin{aligned}Q_1(\mathbf{r},t;\mathbf{r}',t')=&-\sum_{p=1}^{N}i\hat{r}_j(t)\nabla_j\delta(\mathbf{r}-\mathbf{r}^p(t))i\hat{r}_l(t')\\&\times\nabla_l\delta(\mathbf{r}'-\mathbf{r}'^p(t')),\end{aligned} \tag{3.2}$$

$$Q_2(\mathbf{r},t;\mathbf{r}',t')=\sum_{p=1}^{N}i\hat{r}_j(t)\nabla_j\delta(\mathbf{r}-\mathbf{r}^p(t))\delta(\mathbf{r}'-\mathbf{r}'^p(t')),$$

$$Q_3(\mathbf{r},t;\mathbf{r}',t')=-\sum_{p=1}^{N}i\hat{r}_j(t')\nabla_j\delta(\mathbf{r}'-\mathbf{r}^p(t'))\delta(\mathbf{r}-\mathbf{r}^p(t)).$$

After the introduction of the four-dimensional column fields,

$$Q_{\rm a}(1;1')=\begin{bmatrix}Q_0(1;1')\\Q_1(1;1')\\Q_2(1;1')\\Q_3(1;1')\end{bmatrix}, \tag{3.3}$$

where $a=1$, 2, 3, 4, and the 4×4 matrix,

$$\Gamma_{\mathrm{ab}}(1,2,3,4)=\begin{bmatrix} 0 & v(1,3)v(2,4) & 0 & 0\\ v(1,3)v(2,4) & 0 & 0 & 0\\ 0 & 0 & 0 & v(1,4)v(3,2)\\ 0 & 0 & v(1,4)v(3,2) & 0 \end{bmatrix}, \tag{3.4}$$

the whole expression for the GF takes the form

$$\langle Z\rangle_{\mathrm{av}}\{\chi_\alpha,H_a\}=\int\prod_{\alpha=0}^{1}\prod_{a=0}^{3}D\rho_\alpha(1)DQ_a(1;1')\exp\Big\{\tilde W\{\rho_\alpha(1);Q_a(1;1')\}-\frac{\xi_0}{2N}\int d1d2\rho_\alpha(1)U_{\alpha\beta}(1,2)\rho_\beta(2)$$
$$+\int d1\rho_\alpha(1)\chi_\alpha(1)-\frac{\chi^2}{4N^2}\int d1d2d3d4Q_a(1;2)\Gamma_{\mathrm{ab}}(1,2,3,4)Q_{\mathrm{b}}(3;4)+\int d1d2Q_a(1;2)H_a(1;2)\Big\}, \tag{3.5}$$

where the entropy is given by

$$\tilde W\{\rho_\alpha(1);Q_{\mathrm{a}}(1;1')\}=\ln\int\prod_{p=1}^{N}D\mathbf{r}^{(p)}(t)D\hat{\mathbf{r}}^{(p)}(t)\exp\{A_0\{\mathbf{r}^{(p)},\hat{\mathbf{r}}^{(p)}\}\}\prod_{\alpha=0}^{1}\delta\Big[\rho_\alpha(1)-\sum_{p=1}^{N}r_\alpha^{(p)}(1)\delta(\mathbf{r}_1-\mathbf{r}^{(p)}(t))\Big]$$
$$\times\prod_{\mathrm{a}=1}^{4}\delta\Big[Q_a(1;2)-\sum_{p=1}^{N}p_a^{(p)}(1;2)\delta(\mathbf{r}_1-\mathbf{r}^{(p)}(t_1))\delta(\mathbf{r}_2-\mathbf{r}^{(p)}(t_2))\Big]. \tag{3.6}$$

We had used the column operators

$$r_\alpha^{(p)}(1)=\begin{pmatrix}1\\ i\hat r_j^{(p)}(t_1)\nabla_{j,1}\end{pmatrix},$$

$$p_a^{(p)}(1;2)=\begin{pmatrix}1\\ -i\hat r_j^{(p)}(t_1)\nabla_{j,1}i\hat r_l^{(p)}(t_2)\nabla_{l,2}\\ i\hat r_j^{(p)}(t_1)\nabla_{j,1}\\ -i\hat r_j^{(p)}(t_2)\nabla_{j,2}\end{pmatrix} \tag{3.7}$$

and the external field, $H_a(1;2)$, conjugated to $Q_a(1;2)$, has been introduced also.

The two-point collective fields (3.2) have a meaning of the dynamical ''overlaps.'' It is a dynamical generalization of the Parisi ''overlaps'' in a replica space [33]. For example, $Q_0(1;1')$ quantify density-density and $Q_2(1;1')$ response-density overlaps, respectively, between two space-time points. The ''entropy'' (3.6) corresponds to the volume in the dynamical phase space when not only fields $\rho_\alpha(1)$ but also overlaps $Q_a(1;1')$ are given. In a sense the ''entropy'' (3.6) is again the generalization of the entropy for the heteropolymer spanned in a replica space at the given set of ''overlaps'' [29].

B. The saddle-point treatment

Let us introduce the functional $\tilde F\{\psi_\alpha(1);\Phi_a(1;1')\}$ by the functional Fourier transformation,

$$\exp\{\tilde W\{\rho_\alpha(1);Q_a(1;1')\}\}$$
$$=\int\prod_{\alpha=0}^{1}\prod_{a=0}^{3}D\psi_\alpha(1)D\Phi_a(1;1')$$
$$\times\exp\Big\{\tilde F\{\psi_\alpha(1);\Phi_a(1;1')\}+i\int d1\rho_\alpha(1)\psi_\alpha(1)$$
$$+i\int d1d2Q_a(1;2)\Phi_a(1;2)\Big\}. \tag{3.8}$$

After substitution in Eq. (3.5) and integration over ρ_α and $Q_a(1;2)$, one gets

$$\langle Z\rangle_{\mathrm{av}}\{\chi_\alpha,H_a\}=\int\prod_{\alpha=0}^{1}\prod_{a=0}^{3}D\psi_\alpha(1)D\Phi_a(1;1')$$
$$\times\exp\Big\{\tilde F\{\psi_\alpha(1);\Phi_a(1;1')\}$$
$$-\frac{N}{2\xi_0}\int d1d2\psi_\alpha(1)[v^{-1}]_{\alpha\beta}(1,2)\psi_\beta(2)$$
$$-\frac{N}{\chi_0^2}\int d1d2d3d4\Phi_a(1;2)$$
$$\times[\Gamma^{-1}]_{\mathrm{ab}}(1,2,3,4)\Phi_b(3;4)\Big\}, \tag{3.9}$$

where

$$\tilde{F}\{\psi_\alpha(1);\Phi_a(1;1')\}=\ln\int\prod_{p=1}^{N}D\mathbf{r}^{(p)}(t)D\hat{\mathbf{r}}^{(p)}(t)\exp\Bigg\{\sum_{p=1}^{N}A_0[\mathbf{r}^{(p)},\hat{\mathbf{r}}^{(p)}]-i\sum_{p=1}^{N}\int dtr_\alpha^{(p)}(t)[\psi_\alpha(\mathbf{r}^{(p)}(t))+i\chi_\alpha(\mathbf{r}^{(p)}(t))]$$
$$-i\sum_{p=1}^{N}\int dtdt'p_a^{(p)}(t;t')\{\Phi_a[\mathbf{r}^{(p)}(t);\mathbf{r}^{(p)}(t')]+iH_a[\mathbf{r}^{(p)}(t);\mathbf{r}^{(p)}(t')]\}\Bigg\}. \tag{3.10}$$

In order to ensure the extensivity of the whole effective action in Eq. (3.9), we put the variance $\chi^2=\chi_0^2N$ [so that the variance of the whole strength factor in Eq. (2.4) scaled as $N^{-1/2}$ akin to Ref. [28]]. This enables us to represent the GF in a form similar to Eq. (2.20),

$$\langle Z\rangle_{\mathrm{av}}\{\chi_\alpha\}=\int\prod_{\alpha=0}^{1}\prod_{a=1}^{3}D\psi_\alpha(1)D\Phi_a\exp\{-N\tilde{A}[\psi_\alpha,\Phi_a;\chi_\alpha,H_a]\}, \tag{3.11}$$

where

$$\tilde{A}[\psi_\alpha,\Phi_a;\chi_\alpha,H_a]=\frac{1}{2\xi_0}\int dt\int d1d2\,\psi_\alpha(1)[v^{-1}]_{\alpha\beta}(1,2)\psi_\beta(2)+\frac{1}{\chi_0^2}\int d1d2d3d4\,\Phi_a(1;2)[\Gamma^{-1}]_{\mathrm{ab}}(1,2,3,4)\Phi_b(3;4)$$
$$-\frac{1}{N}\ln\int\prod_{p=1}^{N}D\mathbf{r}^{(p)}(t)D\hat{\mathbf{r}}^{(p)}(t)\exp\Bigg\{\sum_{p=1}^{N}A_0[\mathbf{r}^{(p)},\hat{\mathbf{r}}^{(p)}]-i\sum_{p=1}^{N}\int dtr_\alpha^{(p)}(t)[\psi_\alpha(\mathbf{r}^{(p)}(t))+i\chi_\alpha(\mathbf{r}^{(p)}(t))]$$
$$-i\sum_{p=1}^{N}\int dtdt'p_a^{(p)}(t;t')\{\Phi_a[\mathbf{r}^{(p)}(t);\mathbf{r}^{(p)}(t')]+iH_a[\mathbf{r}^{(p)}(t);\mathbf{r}^{(p)}(t')]\}\Bigg\}. \tag{3.12}$$

The resulting SP equation reads

$$\overline{\psi_\alpha}(1)=-\frac{i\xi_0}{N}\int d2v_{\alpha\beta}(1,2)\langle\rho_\beta(2)\rangle_{\mathrm{SP}}, \tag{3.13}$$

$$\overline{\Phi_a}(1)=-\frac{i\chi_0^2}{N}\int d3d4\Gamma_{ab}(1,2,3,4)\langle Q_b(3;4)\rangle_{\mathrm{SP}}, \tag{3.14}$$

where the average $\langle\cdots\rangle_{\mathrm{SP}}$ is calculated with the GF,

$$\langle Z_0\rangle_{\mathrm{av}}\{\chi_\alpha,H_a\}=\int\prod_{p=1}^{N}D\mathbf{r}^{(p)}(t)D\hat{\mathbf{r}}^{(p)}(t)\exp\Bigg\{\sum_{p=1}^{N}A_0[\mathbf{r}^{(p)},\hat{\mathbf{r}}^{(p)}]-i\sum_{p=1}^{N}\int dtr_\alpha^{(p)}(t)[\overline{\psi_\alpha}(\mathbf{r}^{(p)}(t))+i\chi_\alpha(\mathbf{r}^{(p)}(t))]$$
$$-i\sum_{p=1}^{N}\int dtdt'p_a^{(p)}(t;t')\{\overline{\Phi_a}[\mathbf{r}^{(p)}(t);\mathbf{r}^{(p)}(t')]+iH_a[\mathbf{r}^{(p)}(t);\mathbf{r}^{(p)}(t')]\}\Bigg\}. \tag{3.15}$$

Thereby we are left with the GF of a free system which experiences the external mean fields $\overline{\psi_\alpha}+i\chi_\alpha$ and $\overline{\Phi}_a+iH_a$.

C. The self-consistent Hartree approximation

In order to calculate the GF given by Eq. (3.15), we will use the self-consistent Hartree approximation (SCHA). For this approximation we replace the real action by an appropriate Gaussian one in such a way that all terms which include more than two fields $r_j^{(p)}(t)$ or/and $\hat{r}_j^{(p)}(t)$ are written in all possible ways as products of pairs of $r_j^{(p)}(t)$ or $\hat{r}_j^{(p)}(t)$ coupled to self-consistent averages of the remaining fields.

The analogy between the SCHA and the SP approximation at $N\to\infty$ for the special case when the nonquadratic terms in the action are only the functions of the mean-squared displacement $d^2(t-t')=\sum_{p=1}^{N}\langle[\mathbf{r}^{(p)}(t)-\mathbf{r}^{(p)}(t')]^2\rangle/N$ has been proven in Ref. [9]. In our case the action in Eq. (3.15) has a more general form. In Appendix D we show that the SCHA and the *next to the saddle-point approximation* (NSPA) merge and both become exact, if the GF with an arbitrary action can be treated by a steepest-descent approach at $N\to\infty$.

Let us make the Fourier transformation of the mean fields

$$\overline{\psi}_\alpha(\mathbf{r}^{(p)}(t))=\int\frac{d^3k}{(2\pi)^3}\overline{\psi}_\alpha(\mathbf{k})\exp\{i\mathbf{k}\mathbf{r}^{(p)}(t)\}, \tag{3.16}$$

$$\overline{\Phi}_a(\mathbf{r}^{(p)}(t);\mathbf{r}^{(p)}(t'))=\int\frac{d^3k^1d^3k^2}{(2\pi)^6}\overline{\Phi}_a(\mathbf{k}^1,\mathbf{k}^2)\exp\{i\mathbf{k}^1\mathbf{r}^{(p)}(t)+\imath\mathbf{k}^2\mathbf{r}^{(p)}(t')\}, \tag{3.17}$$

and insert it into Eq. (3.15). Then for Eq. (3.15) we use the

Hartree-type action [see Eq. (D24)]. By doing so we put for simplicity the expectation value $\xi_0=0$. It is easy to assure oneselves also that the ''response-response overlap'' $\langle Q_1(1,1')\rangle=0$ (similar to $\langle\hat{\sigma}\hat{\sigma}\rangle=0$ in Ref. [31]). In the curse of the derivation we have used SP equation (3.14) and defined the correlator (or the incoherent scattering function)

$$C(\mathbf{k}^1,t;\mathbf{k}^2,t')=\frac{1}{N}\langle Q_0(\mathbf{k}^1,t;\mathbf{k}^2,t')\rangle \tag{3.18}$$

as well as the response functions

$$G(\mathbf{k}^1,t;\mathbf{k}^2,t')=-\frac{1}{N}\langle Q_3(\mathbf{k}^1,t;\mathbf{k}^2,t')\rangle \quad \text{at } t'<t, \tag{3.19}$$

$$G(\mathbf{k}^2,t;\mathbf{k}^1,t')=\frac{1}{N}\langle Q_2(\mathbf{k}^1,t;\mathbf{k}^2,t')\rangle \quad \text{at } t'>t, \tag{3.20}$$

where $\langle\cdots\rangle$ stands for the averaging with the Hartree type of action. After collection of all terms, the final result (at $\chi_\alpha=0$ and $H_a=0$) then reads

$$\begin{aligned}\langle Z_0\rangle_{\text{av}}\{\overline{\psi_\alpha},\overline{\Phi_a}\}=&\int\prod_{p=1}^{N}D\mathbf{r}^{(p)}(t)D\hat{\mathbf{r}}^{(p)}(t)\\ &\times\exp\Biggl\{\sum_{p=1}^{N}A_0[\mathbf{r}^{(p)},\hat{\mathbf{r}}^{(p)}]\\ &+\int dtdt'\,i\hat{\mathbf{r}}^{(p)}(t)\mathbf{r}^{(p)}(t)\lambda(t,t')\\ &-\int dtdt'\,i\hat{\mathbf{r}}^{(p)}(t)\mathbf{r}^{(p)}(t')\lambda(t,t')\\ &+\int dtdt'\,i\hat{\mathbf{r}}^{(p)}(t)i\hat{\mathbf{r}}^{(p)}(t')\,\eta(t,t')\Biggr\},\end{aligned} \tag{3.21}$$

where

$$\lambda(t,t')=\frac{2}{3}\chi_0^2\int\frac{d^3k}{(2\pi)^3}k^2|v(k)|^2G(\mathbf{k};t,t')C(\mathbf{k};t,t') \tag{3.22}$$

and

$$\eta(t,t')=\frac{1}{3}\chi_0^2\int\frac{d^3k}{(2\pi)^3}k^2|v(k)|^2[C(\mathbf{k};t,t')]^2. \tag{3.23}$$

In Eqs. (3.21)–(3.23) we have restricted ourselves to the homogeneous case,

$$C(\mathbf{k},t;\mathbf{k}',t')=(2\pi)^3\delta(\mathbf{k}+\mathbf{k}'),C(\mathbf{k};t,t'), \tag{3.24}$$

$$G(\mathbf{k},t;\mathbf{k}',t')=(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')G(\mathbf{k};t,t'),$$

for the correlation and response function. The equation of motion for the one-particle correlator

$$\mathcal{P}(t,t')=\frac{1}{3}\sum_{j=1}^{3}\langle r_j^{(p)}(t)r_j^{(p)}(t')\rangle \tag{3.25}$$

and the corresponding response function

$$\mathcal{G}(t,t')=\frac{1}{3}\sum_{j=1}^{3}\langle i\hat{r}_j^{(p)}(t')r_j^{(p)}(t)\rangle \tag{3.26}$$

(which actually does not depend on the particle index p) can be derived from Eq. (3.21) by using the standard techniques [9]. The resulting equations read

$$\begin{aligned}&\left[m_0\frac{\partial^2}{\partial t^2}+\gamma_0\frac{\partial}{\partial t}+\int_{-\infty}^{t}d\tau\lambda(t,\tau)\right]\mathcal{P}(t,t')\\ &\quad-\int_{-\infty}^{t}d\tau\lambda(t,\tau)\mathcal{P}(\tau,t')+\int_{-\infty}^{t'}d\tau\eta(t,\tau)\mathcal{G}(t',\tau)\\ &\quad=-2T\gamma_0\mathcal{G}(t',t)\end{aligned} \tag{3.27}$$

and

$$\begin{aligned}&\left[m_0\frac{\partial^2}{\partial t^2}+\gamma_0\frac{\partial}{\partial t}+\int_{-\infty}^{t}d\tau\lambda(t,\tau)\right]\mathcal{G}(t,t')\\ &\quad-\int_{-\infty}^{t}d\tau\lambda(t,\tau)\mathcal{G}(\tau,t')=-\delta(t-t').\end{aligned} \tag{3.28}$$

Equations (3.27) and (3.28) should be supplemented with the initial conditions $\gamma_0\mathcal{G}(t+0^+,t)=-1$ and $\mathcal{G}(t,t)=0$. By making use of this condition, equipartition $(m_0/3)\Sigma_{j=1}^3\langle\dot{r}_j(t)\dot{r}_j(t)\rangle=T$, causality $\mathcal{G}(t,t')=0$ at $t\leqslant t'$, as well as the condition $(1/3)\Sigma_{j=1}^3\langle\dot{r}_j(t)r_j(t)\rangle=0$, one finds from Eq. (3.27) the following equation:

$$\begin{aligned}&\left[\frac{1}{2}m_0\frac{\partial^2}{\partial t^2}+\int_{-\infty}^{t}d\tau\lambda(t,\tau)\right]\mathcal{P}(t,t)-\int_{-\infty}^{t}d\tau\lambda(t,\tau)\mathcal{P}(\tau,t')\\ &\quad+\int_{-\infty}^{t}d\tau\eta(t,\tau)\mathcal{G}(t,\tau)=2T.\end{aligned} \tag{3.29}$$

Equations (3.27) and (3.28) have the same structure as the Dyson equation (2.42). After the matrices inversions and going to the time domain, Eqs. (2.42) (in the time-translational invariant case) take the form

$$\begin{aligned}&\left[m_0\frac{\partial^2}{\partial t^2}+\gamma_0\frac{\partial}{\partial t}+\mu(0)\right]G_{01}(t,t')-\int_{-\infty}^{t}d\tau\Sigma_{10}(t-\tau)\\ &\quad\times G_{01}(\tau-t')=\delta(t-t'),\end{aligned} \tag{3.30}$$

$$\begin{aligned}&\left[m_0\frac{\partial^2}{\partial t^2}+\gamma_0\frac{\partial}{\partial t}+\mu(0)\right]G_{00}(t,t')-\int_{-\infty}^{t}d\tau\Sigma_{10}(t-\tau)\\ &\quad\times G_{00}(\tau-t')-\int d\tau\Sigma_{11}(t-\tau)G_{10}(\tau-t')\\ &\quad=2T\gamma_0G_{10}(t-t'),\end{aligned} \tag{3.31}$$

where $\mu(0)=\int_0^\infty dt\lambda(t)$ and the RPA-Fourier spectrum

$$S_{01}=\frac{1}{-i\gamma_0\omega-m_0\omega^2+\mu(0)}. \tag{3.32}$$

Equations (3.27) and (3.28) are converted to the Dyson equations (3.30) and (3.31) provided that

$$\begin{gathered}G_{00}(t)=\mathcal{P}(t),\; G_{01}(t)=-\mathcal{G}(t),\\ \Sigma_{10}(t)=\lambda(t),\Sigma_{11}(t)=\eta(t).\end{gathered} \tag{3.33}$$

We can show [19] that the relation

$$-\beta\frac{\partial}{\partial t}\Sigma_{11}(t)=\Sigma_{10}(t)-\Sigma_{01}(t) \tag{3.34}$$

holds, provided that the FDT is satisfied for $G_{\alpha\beta}(t)$. We then have in addition

$$-\beta\frac{\partial}{\partial t}G_{00}(t)=G_{01}(t)-G_{10}(t). \tag{3.35}$$

Bearing Eqs. (3.33) in mind, Eq. (3.34) takes in our case the form (at $t>0$)

$$-\beta\frac{\partial}{\partial t}\eta(t)=\lambda(t). \tag{3.36}$$

The validity of the relationship (3.36) can be checked by replacing Eqs. (3.22) and (3.23) in Eq. (3.36).

The general equations (3.27) and (3.28) are equivalent, after the necessary changes have been made, to the corresponding equations for the *p*-spin system or a particle in the random potential at the large dimension [7–11]. The most important features of these equations are the *glassy dynamical behavior* and the universal *aging regime*. At low temperatures the system tries to minimize the energy and each particle (with a number p) tends to surround itself with other particles which assure the strength parameter $\mu_{\rm pm}<0$. On the other hand, the long-range interaction tries to support other pairs (ij) corresponding to $\mu_{ij}>0$. As a result, the system becomes ''frustrated'' and many local free-energy minima appear.

In the spirit of Refs. [11,34,35] when $t,t'\to\infty$ we have to discriminate between different cases: (i) the *asymptotic regime* when $(t-t')/t\to 0$ and (ii) the *aging regime* when $(t-t')/t'\to O(1)$. The aging regime is much more complicated because the time-translational invariance and FDT are violated. This regime has been extensively investigated both theoretically [8,11,34,35] and by computer simulation [36,37]. In the following, we restrict ourselves only to the asymptotic regime, for the sake of clarity and simplicity, and since the main features will already be visible.

D. The asymptotic regime

This asymptotic regime is characterized by the large time scales, i.e., $t,t'\to\infty$, but keeping the difference $\tau=t-t'$ finite. Under these circumstances we can define

$$\begin{aligned}\mathcal{P}_{\rm as}(\tau)&=\lim_{t'\to\infty}\mathcal{P}(t'+\tau,t'),\\ \mathcal{G}_{\rm as}(\tau)&=\lim_{t'\to\infty}\mathcal{G}(t'+\tau,t').\end{aligned} \tag{3.37}$$

Then the equation for the displacement $\mathcal{D}_{\rm as}=2[\mathcal{P}_{\rm as}(0)-\mathcal{P}_{\rm as}(\tau)]$, response function $\mathcal{G}_{\rm as}(\tau)$, and the static correlator $\mathcal{P}_{\rm as}(0)$ takes correspondingly the forms

$$\begin{aligned}&\left[m_0\frac{\partial^2}{\partial\tau^2}+\gamma_0\frac{\partial}{\partial\tau}+M\right]\mathcal{D}_{\rm as}(\tau)-\int_0^\tau d\tau'\lambda_{\rm as}(\tau-\tau')\mathcal{D}_{\rm as}(\tau')\\ &-\int_0^\infty d\tau'[\lambda_{\rm as}(\tau+\tau')-\lambda_{\rm as}(\tau')]\mathcal{D}_{\rm as}(\tau')\\ &-2\int_0^\infty d\tau'[\eta_{\rm as}(\tau+\tau')-\eta_{\rm as}(\tau')]\mathcal{G}_{\rm as}(\tau')=2T,\end{aligned} \tag{3.38}$$

$$\left[m_0\frac{\partial^2}{\partial\tau^2}+\gamma_0\frac{\partial}{\partial\tau}+M\right]\mathcal{G}_{\rm as}(\tau)-\int_0^\tau d\tau'\lambda_{\rm as}(\tau-\tau')\mathcal{G}_{\rm as}(\tau')=0, \tag{3.39}$$

$$\mathcal{P}_{\rm as}(0)=\frac{1}{M-M_{\rm as}}\left[T-\frac{1}{2}\int_0^\infty d\tau\lambda_{\rm as}\mathcal{D}_{\rm as}(\tau)-\int_0^\infty d\tau\eta_{\rm as}\mathcal{G}_{\rm as}(\tau)\right], \tag{3.40}$$

where

$$M=\lim_{t\to\infty}\int_{-\infty}^t d\tau\lambda(t,\tau), \tag{3.41}$$

$$M_{\rm as}=\int_0^\infty d\tau\lambda_{\rm as}(\tau). \tag{3.42}$$

However, it is also convenient to define the ''anomaly'' $\overline{M}=M-M_{\rm as}$ [11]. Equations (3.38)–(3.40) has been analyzed first in the context of a polymeric manifold in the random media [9,10] and the random-phase sine-Gordon model [38]. The peculiarity of our model is defined by its memory functions $\lambda_{\rm as}(\tau)$ and $\eta_{\rm as}(\tau)$.

For example, let as give an explicit expression for $\eta_{\rm as}(\tau)$. The Gaussian form of the correlator, $C(\tau)=\exp\{-k^2\mathcal{D}_{\rm as}(\tau)/2\}$, leads from Eq. (3.23) to the result

$$\eta_{\rm as}(\tau)=\frac{\chi_0^2\sqrt{\pi}}{6}\frac{1}{\sqrt{\mathcal{D}_{\rm as}(\tau)}}. \tag{3.43}$$

Usually it is assumed that at high temperature FDT holds, i.e.,

$$-\beta\frac{\partial}{\partial\tau}\mathcal{D}_{\rm as}(\tau)=2\mathcal{G}_{\rm as}(\tau) \tag{3.44}$$

and

$$-\beta\frac{\partial}{\partial\tau}\eta_{\rm as}(\tau)=\lambda_{\rm as}(\tau). \tag{3.45}$$

In this case Eqs. (3.38) and (3.39) merge and take a simple form,

$$\left[m_0\frac{\partial^2}{\partial\tau^2}+\gamma_0\frac{\partial}{\partial\tau}+M\right]\mathcal{D}_{\rm as}(\tau)-\int_0^{\tau}d\tau'\lambda_{\rm as}(\tau-\tau')\mathcal{D}_{\rm as}(\tau')=2T. \quad (3.46)$$

It turns out [9,10,37] that the solution which satisfies the FDT is only stable above a critical temperature T_c. For the stability analysis it is convenient to represent Eq. (3.46) in the form

$$\left[m_0\frac{\partial^2}{\partial\tau^2}+\gamma_0\frac{\partial}{\partial\tau}+\overline{M}+M_{\rm as}(\tau)\right]\mathcal{D}_{\rm as}(\tau)+\beta\int_0^{\tau}d\tau'[\eta_{\rm as}(\tau-\tau')-\eta_{\rm as}(\tau)]\frac{\partial}{\partial\tau'}\mathcal{D}_{\rm as}(\tau')=2T, \quad (3.47)$$

where

$$M_{\rm as}(\tau)=\int_{\tau}^{\infty}d\tau'\lambda_{\rm as}(\tau'). \quad (3.48)$$

For $\tau\to\infty$ the stability condition which comes out of Eq. (3.47) reads

$$[\overline{M}+M_{\rm as}(\tau)]\mathcal{D}_{\rm as}(\tau)\leqslant 2T. \quad (3.49)$$

Then the stationary value of the displacement $\mathcal{D}_{\rm as}(\tau\to\infty)=q_0$ reads

$$q_0=\frac{2T}{\overline{\overline{M}}}. \quad (3.50)$$

By taking into account Eqs. (3.44) and (3.45), the stability condition becomes

$$D(q,T)\geqslant 0 \quad (3.51)$$

for $0\leqslant q\leqslant q_0$, where

$$D(q,T)\equiv\left[\left(\frac{\chi_0}{T}\right)^2\frac{\sqrt{\pi}}{12\sqrt{q_0}}-\frac{1}{q_0}\right]q-\left(\frac{\chi_0}{T}\right)^2\frac{\sqrt{\pi}}{12}\sqrt{q}+1. \quad (3.52)$$

The critical values q_c and T_c at which the condition (3.51) first becomes violated is defined by the equations

$$D(q_c,T_c)=0,\qquad D'(q_c,T_c)=0. \quad (3.53)$$

Consequently, Eqs. (3.53) have the simple solution

$$\left(\frac{T_c}{\chi_0}\right)^2=\frac{\sqrt{\pi q_0}}{24}\quad\text{and}\quad q_c=q_0. \quad (3.54)$$

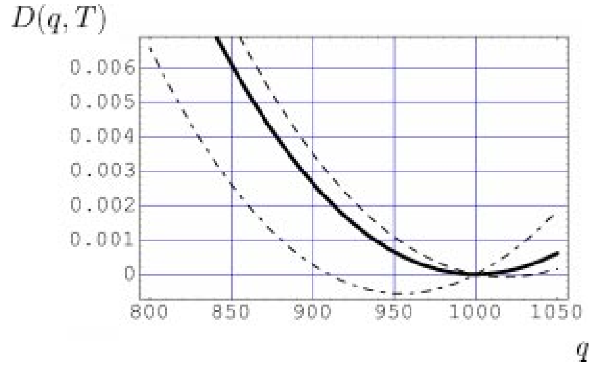

FIG. 3. $D(q,T)$ vs q at $\chi_0=0.1, q_0=10^3$ for different temperatures: (i) full line corresponds to $T=T_c=0.1528$; (ii) dashed line to $T=0.1535$; (iii) dot - dashed line to $T=0.151$.

Figure 3 shows the behavior of $D(q,T)$ in the vicinity of the critical point. It can be seen that the minimum, $q_m\leqslant q_0$, at which $D(q,T)\leqslant 0$ appears continuously, i.e., the instability of the FDT solution, shows up as a second-order phase transition. This is analogous to the dynamics of polymeric manifolds in a medium with the long-range correlation in disorder [10]. In particular, if ''anomaly'' $\overline{M}\to 0$, then $q_0\to\infty$ and $T_c\to\infty$, so in this case the FDT solution is unstable for any finite temperature.

Let us consider the dynamics at the temperature slightly above the critical point: $T=T_c(1+\varepsilon)$, where $0<\varepsilon\ll 1$. For large τ the decomposition

$$\mathcal{D}_{\rm as}(\tau)=q_0-f(\tau) \quad (3.55)$$

is possible, where $f(\tau)\ll q_0$. The substitution of this decomposition into Eq. (3.47) and the expansion up to the second order with respect to $f(\tau)$ yields

$$\varepsilon q_0 f(\tau)+\frac{1}{8}[f(\tau)]^2+\frac{1}{2}\int_0^{\tau}d\tau'[f(\tau-\tau')-f(\tau)]\times\frac{\partial}{\partial\tau'}f(\tau')=0. \quad (3.56)$$

Following Ref. [1], let us make the Laplace transformation $\mathcal{L}[f(\tau)]\equiv\tilde{f}(z)$ and introduce the scaling functions $\hat{\phi}(\hat{z})$ or $\hat{\phi}(\hat{\tau})$ in such a way that

$$\tilde{f}(z)=\frac{c_\varepsilon}{\omega_\varepsilon}\hat{\phi}(\hat{z})\quad\text{or}\quad f(\tau)=c_\varepsilon\hat{\phi}(\hat{\tau}), \quad (3.57)$$

where $\hat{z}=z/\omega_\varepsilon$ and $\hat{\tau}=\tau\omega_\varepsilon$. If $c_\varepsilon=\varepsilon$ and $\omega_\varepsilon=\omega_0\varepsilon^{1/a}$, then one can write Eq. (3.56) in the form

$$q_0\hat{\phi}(\hat{z})-\frac{3}{8}\mathcal{L}\{\hat{\phi}^2(\hat{\tau})\}(\hat{z})+\hat{z}\hat{\phi}^2(\hat{z})=0 \quad (3.58)$$

[see Eq. (2.68b) of Ref. [1]].

In the critical regime $\hat{z}\gg 1$ (or $\hat{\tau}\ll 1$), the solution of Eq. (3.58) has a form $\hat{\phi}(\hat{\tau})\propto\hat{\tau}^{-a}$. In this limit the first term in Eq. (3.58) is dropped out and the exponent is defined by the equation

$$\frac{\Gamma^2(1-a)}{\Gamma(1-2a)}=\frac{3}{4}. \tag{3.59}$$

The solution of Eq. (3.59) gives $a=0.304\,65$. In the opposite limit $\hat{z}\ll 1$ (or $\hat{\tau}\gg 1$), the last term in Eq. (3.58) can be neglected. In this case the solution has the form $\hat{\phi}(\hat{\tau}) \propto A_\varepsilon \hat{\tau}^{-a}\exp\{-\lambda\hat{\tau}\}$, where $A_\varepsilon=8\varepsilon q_0\Gamma(1-a)2^{(1-2a)}/3\Gamma(1-2a)\lambda^2$. As a result, the overall scaling reads

$$\mathcal{D}_{\text{as}}(\tau)=\begin{cases} q_0-\dfrac{c_\varepsilon}{(\omega_\varepsilon\tau)^a} & \text{at } \omega_\varepsilon\tau\ll 1\\[2ex] q_0-\dfrac{A_\varepsilon}{(\omega_\varepsilon\tau)^a}\exp\{-\lambda(\omega_\varepsilon\tau)\} & \text{at } \omega_\varepsilon\tau\gg 1,\end{cases} \tag{3.60}$$

where λ is some constant.

At $T<T_c$, the FDT is violated for the large time separation τ and the aging regime arises. It should be mentioned that the asymptotic regime cannot be decoupled from the aging one [11,35]. In actual fact, the ''anomaly'' $\overline{M}$ in the asymptotic equations (3.38)–(3.40) strictly speaking can be calculated only from the aging regime. Because of the distinct aim of this paper, we are not going to discuss the aging regime here, expecting to return to it in a later publication.

IV. CONCLUSION

In the present paper we have considered the dynamics of two models with the long-range repulsive interaction. The interaction potential was designed in a way to enable the saddle-point treatment as well as a fluctuation expansion.

For the pure model we have derived Eq. (2.53) for the full correlation matrix $G_{\alpha\beta}(\mathbf{k},\omega)$ in the one-loop approximation, which has an explicit solution [see Eqs. (2.57)–(2.59)]. This solution has a ''boring behavior'' at $\omega\to 0$ which manifests the absence of the glass dynamics. The physical background of this stems from the fact that the potential is much too soft and the ''cage effect'' is completely missing.

This conclusion is in accordance with the interacting particles statistical thermodynamics analysis, which was given in Ref. [12]. It was shown there that for the infinite range interaction potential, which allows a well-defined saddle-point treatment, the glassy phase is simply suppressed.

On the other hand, the same model but with a randomly distributed strength of interaction (the ''random-bond model'') leads to the continuous glass transition. This type of transition is also the case for the polymeric manifolds in the disordered medium with long-range correlation [10] as well as for the *p*-spin interaction spin-glass model at the large external field [3,6]. It would be also interesting to investigate the more realistic ''random sequence model'' in which each particle carries a random ''charge.''

Qualitatively, the same glassy behavior has been found in the pure spin models with the deterministic but very rapidly oscillating coupling between variables [39–42]. It was assumed that the effective quenched disorder is ''self-induced'' [8,39]. This means that because of the slow dynamics, some degrees of freedom freeze and play the role of the effectively quenched disorder.

In conclusion, the glass transition in the pure systems of the interacting particles, where the disorder is actually ''self-induced,'' goes beyond the mean-field level [12]. This appears too difficult to implement in the present context, because it implies the consideration of the short-range interaction potential as well as activated processes.

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APPENDIX A: CALCULATION OF THE FOUR-POINT RPA-CORRELATION MATRIX

In the full analogy with Eq. (2.26), the expression for $S^{(4)}_{\alpha\beta\gamma\delta}(1,2,3,4)$ reads

$$S^{(4)}_{\alpha\beta\gamma\delta}(1,2,3,4) = \lim_{\overline{\psi}_\alpha+i\chi_\alpha\to 0}\left[\frac{\delta}{N^2\delta\chi_\beta(2)\delta\chi_\gamma(3)\delta\chi_\delta(4)}\langle\rho_\alpha(1)\rangle_{\text{SP}}\right]. \tag{A1}$$

The expansion of the $\langle\rho_\alpha(1)\rangle_{\text{SP}}$ up to the third order with respect to the mean field $\overline{\psi}_\alpha+i\chi_\alpha$ can be easily obtained from Eq. (2.25),

$$\begin{aligned}\langle\rho_\alpha(1)\rangle_{\text{SP}}=&\langle\rho_\alpha(1)\rangle_0+\int d2\langle\Delta\rho_\alpha(1)\Delta\rho_\beta(2)\rangle_0[\chi_\beta(2)\\ &-i\overline{\psi}_\beta(2)]+\frac{1}{2!}\int d2d3\langle\Delta\rho_\alpha(1)\\ &\times\Delta\rho_\beta(2)\Delta\rho_\gamma(3)\rangle_0[\chi_\beta(2)-i\overline{\psi}_\beta(2)][\chi_\gamma(3)\\ &-i\overline{\psi}_\gamma(3)]+\frac{1}{3!}\int d2d3d4\langle\Delta\rho_\alpha(1)\Delta\rho_\beta(2)\\ &\times\Delta\rho_\gamma(3)\Delta\rho_\delta(4)\rangle_0[\chi_\beta(2)\\ &-i\overline{\psi}_\beta(2)][\chi_\gamma(3)-i\overline{\psi}_\gamma(3)][\chi_\delta(4)-i\overline{\psi}_\delta(4)].\end{aligned} \tag{A2}$$

By using the SP equation (2.24) and after threefold differentiation with respect to $\chi_\alpha(1)$ [see Eq. (A1)] we find

$$\begin{aligned}S^{(4)}_{\alpha\beta\gamma\delta}(1,2,3,4)=&F^{(4)}_{\overline{\alpha\beta\gamma\delta}}(\overline{1},\overline{2},\overline{3},\overline{4})\{[\hat{1}+\mu\hat{v}\hat{F}^{(2)}]^{-1}\}_{\overline{\alpha}\alpha}(\overline{1},1)\\ &\times\{[\hat{1}+\mu\hat{v}\hat{F}^{(2)}]^{-1}\}_{\overline{\beta}\beta}(\overline{2},2)\\ &\times\{[\hat{1}+\mu\hat{v}\hat{F}^{(2)}]^{-1}\}_{\overline{\gamma}\gamma}(\overline{3},3)\\ &\times\{[\hat{1}+\mu\hat{v}\hat{F}^{(2)}]^{-1}\}_{\overline{\delta}\delta}(\overline{4},4),\end{aligned} \tag{A3}$$

where the four-point free system correlation matrix

$$F^{(4)}_{\alpha\beta\gamma\delta}(1,2,3,4)=\frac{1}{N^2}\langle\Delta\rho_\alpha(1)\Delta\rho_\beta(2)\Delta\rho_\gamma(3)\Delta\rho_\delta(4)\rangle_0. \tag{A4}$$

In Eq. (A3) we imply the summation (integration) over the barred indices (barred space-time variables). When deriving Eq. (A3) we have also kept in mind that $\langle\Delta\rho_\alpha(1)\Delta\rho_\beta(2)\rangle \propto N$ and $\langle\Delta\rho_\alpha(1)\Delta\rho_\beta(2)\Delta\rho_\gamma(3)\Delta\rho_\delta(4)\rangle_0\propto N^2$, etc. The fact that the matrix $F^{(4)}_{\alpha\beta\gamma\delta}(1,2,3,4)$ is symmetrical with respect to simultaneous permutations of Greek indices and space-time arguments as well as Eq. (2.27) have been used.

It is easy to show that $F^{(4)}_{\alpha\beta\gamma\delta}(1,2,3,4)$ is factorized,

$$F^{(4)}_{\alpha\beta\gamma\delta}(1,2,3,4)=F^{(2)}_{\alpha\beta}(1,2)F^{(2)}_{\gamma\delta}(3,4)+F^{(2)}_{\alpha\gamma}(1,3)F^{(2)}_{\beta\delta}(2,3) + F^{(2)}_{\alpha\delta}(1,4)F^{(2)}_{\beta\gamma}(2,3). \tag{A5}$$

On the other side it is instructive to check that even in this case $S^{(4)}_{\alpha\beta\gamma\delta}(1,2,3,4)$ cannot be factorized.

APPENDIX B: CALCULATION OF THE VERTEX MATRIX $K_{\alpha\beta}(1,2)$

The substitution of Eg. (A3) into Eq. (2.51) after straightforward algebra yields

$$\begin{aligned}K_{\alpha\beta}(1,2)=&\{[(2\mu)^{-1}\hat{v}^{-1}+\hat{F}]^{-1}-[\mu^{-1}\hat{v}^{-1}+\hat{F}]^{-1}\}_{\overline{\beta\alpha}}\\&\times(\overline{2},\overline{1})\{F_{\overline{\alpha\beta}}(\overline{1},\overline{2})F_{\overline{\gamma\delta}}(\overline{3},\overline{4})\\&+F_{\overline{\alpha\gamma}}(\overline{1},\overline{3})F_{\overline{\beta\delta}}(\overline{2},\overline{4})+F_{\overline{\alpha\delta}}(\overline{1},\overline{4})F_{\overline{\beta\gamma}}(\overline{2},\overline{3})\}\\&\times\{[\overline{1}+2\mu\overline{vF}]^{-1}\}_{\overline{\gamma}\alpha}(\overline{3},1)\\&\times\{[\overline{1}+2\mu\hat{v}\hat{F}]^{-1}\}_{\overline{\delta}\beta}(\overline{4},2),\end{aligned} \tag{B1}$$

where as before for the repeated barred indices (variables) the summation (integration) is implied. For the time-space-translational invariant case the respective Fourier transformation leads to the result

$$\begin{aligned}K_{\alpha\beta}(\mathbf{k},\omega)=&\{IF_{\overline{\gamma\delta}}(\mathbf{k},\omega)+\{[(2\mu)^{-1}\hat{v}^{-1}+\hat{F}]^{-1}\\&-[\mu^{-1}\hat{v}^{-1}+\hat{F}]^{-1}\}_{\overline{\beta\alpha}}(-\mathbf{k},-\omega)F_{\overline{\alpha\gamma}}(-\mathbf{k},-\omega)\\&\times F_{\overline{\beta\delta}}(\mathbf{k},\omega)+\{[(2\mu)^{-1}\hat{v}^{-1}+\hat{F}]^{-1}\\&-[\mu^{-1}\hat{v}^{-1}+\hat{F}]^{-1}\}_{\overline{\beta\alpha}}(\mathbf{k},\omega)F_{\overline{\alpha\delta}}(\mathbf{k},\omega)\\&\times F_{\overline{\beta\gamma}}(-\mathbf{k},-\omega)\}\{[\overline{1}+2\mu\hat{v}\hat{F}]^{-1}\}_{\overline{\gamma}\alpha}\\&\times(-\mathbf{k},-\omega)\{[\overline{1}+2\mu\overline{vF}]^{-1}\}_{\overline{\delta}\beta}(\mathbf{k},\omega),\end{aligned} \tag{B2}$$

where the trace

$$\begin{aligned}I=\int\frac{d^3q\,d\omega}{(2\pi)^4}&\{[\hat{1}+(2\mu)^{-1}\hat{F}^{-1}\hat{v}^{-1}]^{-1}\\&-[\hat{1}+\mu^{-1}\hat{F}^{-1}\hat{v}^{-1}]^{-1}\}_{\overline{\alpha\alpha}}(\mathbf{q},\omega).\end{aligned} \tag{B3}$$

With the correlation matrix $\hat{F}$ given by Eqs. (2.56), by doing integration over ω one can check that the trace $I=0$. This gives finally

$$K_{\alpha\beta}(\mathbf{k},\omega)=L_{\alpha\beta}(\mathbf{k},\omega)+L_{\alpha\beta}(-\mathbf{k},-\omega), \tag{B4}$$

where

$$\begin{aligned}L_{\alpha\beta}(\mathbf{k},\omega)=&\{[\overline{1}+2\mu\hat{v}\hat{F}]^{-1}\}_{\alpha\overline{\gamma}}(\mathbf{k},\omega)F_{\overline{\gamma\beta}}(\mathbf{k},\omega)\\&\times\{[\hat{1}+(2\mu)^{-1}\hat{F}^{-1}\hat{v}^{-1}]^{-1}\\&-[\hat{1}+\mu^{-1}\hat{F}^{-1}\hat{v}^{-1}]^{-1}\}_{\overline{\beta\delta}}(\mathbf{q},\omega)\\&\times\{[\overline{1}+2\mu\hat{v}\hat{F}]^{-1}\}_{\alpha\overline{\gamma}}(\mathbf{k},\omega).\end{aligned} \tag{B5}$$

APPENDIX C: THE MCT FOR THE GENERALIZED KAC POTENTIAL

In this case the direct correlation function $c(\mathbf{r}) = -\beta V(\mathbf{r})$ and its Fourier transformation take the scaling form

$$c(\mathbf{k})=-\beta f\left(\frac{\mathbf{k}}{\kappa}\right). \tag{C1}$$

Let us insert this expression into the MCT-memory kernel [see Eq. (3.32) in [1]]. It is reasonable then to rescale the integration variables in the memory kernel, $\mathbf{k}\to\kappa\mathbf{k}$, $\mathbf{p}\to\kappa\mathbf{p}$, as well as to put for the external wave vector $\mathbf{q}=\kappa\mathbf{q}_0$, where $\mathbf{q}_0$ is some reference wave vector. The last scaling means that in the MF limit an experiment probes a very small wave vector: $\mathbf{q}\to 0$. The resulting scaling of the memory kernel, $m(\mathbf{q},t)$, reads

$$\begin{aligned}m(\kappa\mathbf{q}_0,t)=&\kappa^d\tilde{S}(\mathbf{q}_0)\frac{\rho_0}{2}\int\frac{d\mathbf{k}d\mathbf{p}}{(2\pi)^{2d}}\delta^{(d)}(\mathbf{k}+\mathbf{p}-\mathbf{q}_0)\\&\times\frac{\{e^L(\mathbf{q})\beta[\mathbf{k}f(\mathbf{k})+\mathbf{p}f(\mathbf{p})]\}^2}{q_0^2}\tilde{S}(\mathbf{k},t)\tilde{S}(\mathbf{p},t),\end{aligned} \tag{C2}$$

where we have taken into account the scaling form of the correlator: $S(\mathbf{k},t)=\tilde{S}(\mathbf{k}/\kappa;t)$. Thus we finally arrive at the scaling $m(\kappa\mathbf{q}_0,t)\propto\kappa^d\to 0$ and the glass transition dies out.

APPENDIX D: THE ANALOGY BETWEEN THE SCHA AND NSPA

Let us prove that the SCHA becomes exact for the GF given by Eq. (3.15) in the limit $N\to\infty$. We will consider even a more general GF,

$$\begin{aligned}Z\{\chi_\alpha\}=&\int\prod_{p=1}^{N}\prod_{\alpha=0,1}Dx_\alpha^{(p)}(1)\\&\times\exp\left\{-\frac{1}{2}\sum_{p=1}^{N}\int d1d2x_\alpha^{(p)}(1)A_{\alpha\beta}(1,2)x_\beta^{(p)}(2)\right.\\&\left.+\sum_{p=1}^{N}W[x_\alpha^{(p)}]+\sum_{p=1}^{N}\int d1x_\alpha^{(p)}(1)\chi_\alpha(1)\right\},\end{aligned} \tag{D1}$$

where we have used the short-hand notations

$$x_\alpha^{(p)}(1)=\begin{pmatrix} r_j(t) \\ i\hat{r}_j(t)\end{pmatrix}, \tag{D2}$$

and "1" embraces Cartesian indices as well as the time variable: $1\equiv\{i,j,k;t\}$. In Eq. (D1), $W[x_\alpha^{(p)}]$ is an arbitrary functional of $x_\alpha^{(p)}$.

Instead of the exact action functional in Eq. (D1), we consider now the trial one which has a Gaussian form,

$$S[x_\alpha^{(p)}(1)]=\sum_{p=1}^{N}\left\{\frac{1}{2}\int d1d2x_\alpha^{(p)}(1)A_{\alpha\beta}(1,2)x_\beta^{(p)}(2)-\int d1d2x_\alpha^{(p)}(1)\Gamma_{\alpha\beta}(1,2)x_\beta^{(p)}(2)-\int d1L_\alpha(1)x_\alpha^{(p)}(1)\right\}. \tag{D3}$$

Let us look for the "best" coefficients $\Gamma_{\alpha\beta}(1,2)$ and $L_\alpha(1)$ in a sense that the exact "free energy" $F[\chi_\alpha]=-\ln Z\{\chi_\alpha\}$ tends to the trial one $F_0[\chi_\alpha]=-\ln\int\Pi Dx_\alpha^{(p)}\exp\{-S[x_\alpha^{(p)}]\}$, i.e.,

$$F[\chi_\alpha]\to F_0[\chi_\alpha], \tag{D4}$$

and both become exact at $N\to\infty$.

We can show that the property (D3) is satisfied by $\Gamma_{\alpha\beta}$ and L_α, which are obtained by extremization of the functional

$$\Phi\{\Gamma_{\alpha\beta},L_\alpha\}=-\ln\int\prod_{p=1}^{N}\prod_{\alpha=0,1}Dx_\alpha^{(p)}(1)\exp\{-S[x_\alpha^{(p)}]\}+\sum_{p=1}^{N}\left\{\int d1d2\Gamma_{\alpha\beta}(1,2)\langle x_\alpha^{(p)}(1)x_\beta^{(p)}(2)\rangle_s+\int d1[L_\alpha(1)-\chi_\alpha]\langle x_\alpha^{(p)}(1)\rangle_s-\langle W[x_\alpha^{(p)}(1)]\rangle_s\right\}, \tag{D5}$$

where we use the notations

$$\langle\cdots\rangle_s=\frac{\int\prod_{p=1}^{N}\prod_{\alpha=0,1}Dx_\alpha^{(p)}(1)\cdots\exp\{-S[x_\alpha^{(p)}]\}}{\int\prod_{p=1}^{N}\prod_{\alpha=0,1}Dx_\alpha^{(p)}(1)\exp\{-S[x_\alpha^{(p)}]\}}. \tag{D6}$$

The extremization conditions read

$$\frac{\delta}{\delta\Gamma_{\alpha\gamma}(1,2)}\Phi=0,\qquad \frac{\delta}{\delta L_\alpha(1)}\Phi=0. \tag{D7}$$

The variations in Eq. (D6) can be done directly. During the calculation the generalized Wick's theorem [22] should also be taken into account. Namely, because the averaging (D5) is simply the Gaussian integral, Wick's theorem yields

$$\langle x_\alpha^{(p)}(1)W[x_\alpha^{(p)}]\rangle_s=\langle x_\alpha^{(p)}(1)\rangle_s\langle W[x_\alpha^{(p)}]\rangle_s+\int d2\langle\Delta x_\alpha^{(p)}(1)\Delta x_\beta^{(p)}(2)\rangle_s\times\left\langle\frac{\delta}{\delta x_\beta^{(p)}(2)}W[x_\alpha^{(p)}]\right\rangle_s, \tag{D8}$$

where $\Delta x_\alpha^{(p)}(1)\equiv x_\alpha^{(p)}(1)-\langle x_\alpha^{(p)}(1)\rangle_s$. After straightforward calculation we find

$$\Gamma_{\alpha\gamma}(1,2)=\frac{1}{2}\left\langle\frac{\delta^2}{\delta x_\alpha^{(p)}(1)\delta x_\beta^{(p)}(2)}W[x_\alpha^{(p)}]\right\rangle_s \tag{D9}$$

and

$$L_\alpha(1)=\int d2[A_{\alpha\beta}(1,2)-\Gamma_{\alpha\beta}(1,2)]\langle x_\alpha^{(p)}(2)\rangle_s. \tag{D10}$$

Then equations for the two moments take the form

$$\langle x_\alpha^{(p)}(1)\rangle_s=\int d2[A^{-1}]_{\alpha\beta}(1,2)\left[\left\langle\frac{\delta}{\delta x_\beta^{(p)}(2)}W[x_\alpha^{(p)}]\right\rangle_s+\chi_\beta(2)\right] \tag{D11}$$

and

$$\langle\Delta x_\alpha^{(p)}(1)\Delta x_\beta^{(p)}(2)\rangle_s=\{[\hat{A}-2\hat{\Gamma}]^{-1}\}_{\alpha\beta}(1,2), \tag{D12}$$

where $\hat{A}$ and $\hat{\Gamma}$ stand for the corresponding 2×2 matrices.

On the other side, the saddle-point (SP) treatment of Eq. (D1) at $N\to\infty$ yields

$$-\int d2A_{\alpha\beta}(1,2)\overline{x_\beta^{(p)}}(2)+\left.\frac{\delta W}{\delta x_\alpha^{(p)}(1)}\right|_{x_\alpha=\overline{x}_\alpha}+\chi_\alpha(1)=0 \tag{D13}$$

and

$$\langle\Delta x_\alpha^{(p)}(1)\Delta x_\beta^{(p)}(2)\rangle_{\rm SP}=\{[\hat{A}-2\hat{B}]^{-1}\}_{\alpha\beta}(1,2), \tag{D14}$$

where

$$B_{\alpha\beta}(1,2)=\left.\frac{1}{2}\frac{\delta^2W}{\delta x_\alpha^{(p)}(1)\delta x_\beta^{(p)}(2)}\right|_{x_\alpha=\overline{x}_\alpha} \tag{D15}$$

and $\overline{x_\alpha^{(p)}}(1)$ stands for the field in SP.

In order to show the analogy between Eqs. (D11) and (D12) and Eqs. (D13) and (D14), let us make the functional Fourier transformation

$$\exp\{K[y_\alpha^{(p)}(1)]\} = \int Dx_\alpha^{(p)}(1)\exp\left\{W[x_\alpha^{(p)}(1)] - i\int d1 x_\alpha^{(p)}(1) y_\alpha^{(p)}(1)\right\} \tag{D16}$$

and its inversion

$$\exp\{W[x_\alpha^{(p)}(1)]\} = \int Dy_\alpha^{(p)}(1)\exp\left\{K[y_\alpha^{(p)}(1)] + i\int d1 x_\alpha^{(p)}(1) y_\alpha^{(p)}(1)\right\}. \tag{D17}$$

Then Eqs. (D13) and (D14) can be written as

$$\overline{x_\alpha^{(p)}}(1) = \int d2 [A^{-1}]_{\alpha\beta}(1,2)[i\langle y_\beta^{(p)}(2)\rangle_{\mathrm{SP}} + \chi_\beta(2)] \tag{D18}$$

and

$$\langle \Delta x_\alpha^{(p)}(1)\Delta x_\beta^{(p)}(2)\rangle_{\mathrm{SP}} = \{[\hat{A} + \|\langle \Delta y \Delta y\rangle_{\mathrm{SP}}\|]^{-1}\}_{\alpha\beta}(1,2), \tag{D19}$$

where the correlation matrix

$$\|\langle \Delta y \Delta y\rangle_{\mathrm{SP}}\| = \langle \Delta y_\alpha^{(p)}(1)\Delta y_\beta^{(p)}(2)\rangle_{\mathrm{SP}} \tag{D20}$$

and

$$\langle \cdots \rangle_{\mathrm{SP}} = \frac{\int Dy_\alpha^{(p)} \cdots \exp\left\{K[y_\alpha^{(p)}] + i\int d1 \overline{x_\alpha^{(p)}}(1) y_\alpha^{(p)}(1)\right\}}{\int Dy_\alpha^{(p)} \exp\left\{K[y_\alpha^{(p)}] + i\int d1 \overline{x_\alpha^{(p)}}(1) y_\alpha^{(p)}(1)\right\}}. \tag{D21}$$

At $N\to\infty$ by making use of Eq. (D17), one can immediately see that

$$\left\langle \frac{\delta}{\delta x_\alpha^{(p)}(1)} W[x_\alpha^{(p)}]\right\rangle_s \to i\langle y_\alpha^{(p)}(1)\rangle_{\mathrm{SP}} \tag{D22}$$

and

$$\left\langle \frac{\delta^2}{\delta x_\alpha^{(p)}(1)\,\delta x_\beta^{(p)}(2)} W[x_\alpha^{(p)}]\right\rangle_s \to -\langle \Delta y_\alpha^{(p)}(1)\Delta y_\beta^{(p)}(2)\rangle_{\mathrm{SP}}, \tag{D23}$$

and the SCHA exactly corresponds to the NSPA. For the case which was treated in Sec. III C, $\langle x_\alpha^{(p)}(1)\rangle_s = 0$ and the Hartree-type action (D3) cast the form

$$S[x_\alpha^{(p)}(1)] = \sum_{p=1}^{N}\left\{\frac{1}{2}\int d1d2\, x_\alpha^{(p)}(1)A_{\alpha\beta}(1,2)x_\beta^{(p)}(2) - \frac{1}{2}\int d1d2\left\langle \frac{\delta^2}{\delta x_\alpha^{(p)}(1)\,\delta x_\beta^{(p)}(2)} W[x_\alpha^{(p)}]\right\rangle_s \times x_\alpha^{(p)}(1)x_\beta^{(p)}(2)\right\}. \tag{D24}$$

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